

CHEMISTRY 047

STUDY PACKAGE

This material is intended as a review of skills you once learned.

PREPARING TO WRITE
THE
ASSESSMENT

SOLVING FORMULAE (PART A)

INTRODUCTION

- A **formula** is an equation with letters. Solving a formula involves manipulating it, so that a particular letter is "isolated" in the formula. The techniques used in manipulating formulae are similar to those previously discussed in solving equations.

FORMULAE WITH ONE TERM ON EACH SIDE

- When solving a formula, first locate the letter that you are asked to solve for. Then, "isolate" the letter in the same way as solving equations.

e.g. $\frac{2x}{3} = \frac{1}{5}$

$$x = \frac{1(3)}{5(2)}$$

$$x = \frac{3}{10}$$

e.g. $\frac{ax}{b} = \frac{2}{c}$, solve for x

$$x = \frac{2b}{ac}$$

e.g. Solve $F = \frac{mv^2}{r}$, for m

$$\frac{Fr}{v^2} = m$$

e.g. Solve $\frac{a}{b} = \frac{c}{d}$, for d

$$\frac{b}{a} = \frac{d}{c}$$

$$\frac{bc}{a} = d$$

e.g. If Force (F) equals mass (m) times acceleration (a), what is the mass?

$$F = ma$$

$$\frac{F}{a} = m$$

The mass equals the force divided by the acceleration.

e.g. The volume (v) of a cylinder equals pi (π) times the radius squared times the height. What is the height?

$$v = \pi r^2 h$$

$$\frac{v}{\pi r^2} = h$$

The height equals the volume divided by the product of π and the radius squared.

FORMULAE WITH BRACKETS

- When solving a formula, a bracket can be thought of as one unit. Note that the bracket $(a + b)$ behaves in an identical manner to the letter x in the examples below.

e.g. $A = \frac{1}{2}hx$, for h

$$\frac{2A}{x} = h$$

e.g. $A = \frac{1}{2}h(a + b)$, for h

$$\frac{2A}{(a + b)} = h \text{ or } \frac{2A}{a + b} = h$$

- When solving for a letter which is inside a bracket, solve for (or isolate) the bracket first. Then a term which is added will be subtracted on the other side of the equation.

e.g. $A = \frac{1}{2}h(a + b)$, for a

$$\frac{2A}{h} = a + b$$

$$\frac{2A}{h} - b = a \text{ or } \frac{2A - bh}{h} = a$$

e.g. $\frac{t(V_1 + V_2)}{2} = S$, for V_1

$$V_1 + V_2 = \frac{2S}{t}$$

$$V_1 = \frac{2S}{t} - V_2 \text{ or } V_1 = \frac{2S - V_2 t}{t}$$

EXERCISE I

Solve:

- | | | |
|--|--|--|
| 1. a) $A = lw$, for w | b) $\frac{V}{R} = I$, for V | c) $\frac{W}{I} = E$, for I |
| d) $I = prt$, for t | e) $E = mc^2$, for m | f) $\frac{A}{\pi} = r^2$, for A |
| g) $V = \frac{4}{3}\pi r^3$, for π | h) $F = \frac{mv^2}{r}$, for r | i) $V = \frac{1}{3}\pi r^2 h$, for h |
| j) $\frac{a}{x} = \frac{c}{2}$, for x | k) $A = \frac{1}{2}bh$, for b | l) $\frac{w_1}{w_2} = \frac{d_1}{d_2}$, for d_2 |
| m) $F = \frac{km_1 m_2}{r^2}$, for k | n) $\frac{E}{e} = \frac{R + r}{r}$, for e | |
-
- | | |
|---|---|
| 2. a) $A = \frac{1}{2}h(a + b)$, for h | b) $A = \frac{1}{2}h(c + d)$, for d |
| c) $C = \frac{5}{9}(F - 32)$, for F | d) $s = \frac{t(V_1 + V_2)}{2}$, for t |
| e) $s = \frac{t(V_1 + V_2)}{2}$, for V_2 | f) $s = \frac{H}{m(t_1 - t_2)}$, for H |

RULES FOR WORKING WITH POSITIVE AND NEGATIVE NUMBERS

★★★ Think of “combining” numbers rather than **adding or subtracting** them.

RULE I: If the signs are the same, add the numbers, and use the same sign.

RULE II: If the signs are different, ignore the signs, subtract the smaller number from the larger and use the larger number’s sign.

Examples

- a) $-5 + 2$ The signs are different, so subtract the numbers
($5 - 2 = 3$) and use the sign of the larger (-5)
 $-5 + 2 = -3$
- b) $2 - 9$ Again, the signs are different, so subtract the numbers
($9 - 2 = 7$) and use the sign of the larger
(the 9 which is negative)
 $2 - 9 = -7$
- c) $-2 - 3$ The signs are the same (both negative), so add the numbers
($2 + 3 = 5$) and use the same sign ($-$)
 $-2 - 3 = -5$
- d) $3 - (-5)$ Use the rules for multiplication:
negative times negative is a positive, so
 $3 - (-5) = 3 + 5 = 8$
- e) $-5 - (-6) = -5 + 6 = 1$

RULE III: When **multiplying or dividing**, if the signs are the same, the answer is positive.

RULE IV: When **multiplying or dividing**, if the signs are different, the answer is negative.

Examples

a) $(-5)(-5) = 25$ The signs are **the same**, so the answer is **positive**.

b) $(-3)(4) = -12$ The signs are **different**, so the answer is **negative**.

c) $(-2)(1)(-6)(3)$
 $= (-2)(-6)(3)$
 $= (12)(3)$
 $= 36$

d) $\frac{-50}{10}$ or $(-50) \div (10) = -5$ The signs are **different**,
so the answer is **negative**.

e) $\frac{-81}{-9}$ or $(-81) \div (-9) = 9$ The signs are the **same**,
so the answer is **positive**.

EXERCISE II

1. $-3 + 4$

2. $8 - 12$

3. $-6 + 7$

4. $-4 - 8$

5. $5 + 8$

6. $10 - 6$

7. $-12 + 28$

8. $14 - 38$

9. $-101 + 33$

10. $(14)(-6)$ or $(14) \times (-6)$

11. $(5)(-8)$ or $(5) \times (-8)$

12. $(-20)(-3)$ or $(-20) \times (-3)$

13. $(14) \div (-2)$

14. $\frac{-36}{-6}$

15. $\frac{-42}{6}$

16. $-7 - (-10)$

17. $-12 + 108$

18. $15 \div (-3)$

19. $-123 - (-10)$

20. $(27) \div (-3)$

21. $(13)(8)$

22. $(-4)(-6)$

23. $-13 + 62$

24. $-94 - (-37)$

25. $(-48) \div (8)$

26. $(-16) \div (-4)$

27. $-\frac{1}{2} + \frac{3}{4}$

28. $-\frac{3}{5} - \frac{7}{8}$

29. $-1.25 + 8.31$

30. $\left(-\frac{2}{3}\right)\left(\frac{1}{4}\right)$ or $\left(-\frac{2}{3}\right) \times \left(\frac{1}{4}\right)$

31. $\left(-5\frac{1}{2}\right)\left(-2\frac{3}{7}\right)$ **or** $\left(-5\frac{1}{2}\right)\times\left(-2\frac{3}{7}\right)$

32. $(3.14)(-2.17)$ **or** $(3.14)\times(-2.17)$

33. $-7.81 - (-3.16)$

34. $-2.5 - 4.8$

35. $(-4.8) \div (-2.2)$

36. $\frac{1}{8} - \frac{3}{4}$

37. $(13.2)(-14.3)$

38. $(-2.6)(-1.8)$

39. $\left(-4\frac{1}{2}\right)\left(2\frac{1}{2}\right)$

40. $(-3.2)(-4.1)$

41. $3\frac{1}{5} - 7\frac{2}{3}$

42. $-2\frac{2}{3} - \left(-3\frac{1}{3}\right)$

43. $(4.68)(-2.13)$

44. $-3\frac{7}{8} - 2\frac{1}{3}$

45. $\left(5\frac{2}{5}\right) \div \left(-2\frac{2}{3}\right)$

46. $\left(1\frac{1}{2}\right) - \left(-2\frac{1}{4}\right)$

47. $(4.3)(-2.4)$

48. $(-1.23)(-2.34)$

49. $4\frac{1}{8} - \left(-2\frac{2}{7}\right)$

50. $-3\frac{8}{13} - 2\frac{4}{5}$

51. $\left(-3\frac{1}{2}\right) \div \left(-8\frac{1}{4}\right)$

52. $-4 + 8$

53. $\frac{1}{4} - \frac{9}{10}$

54. $(-3.25) \div (-0.25)$

SUBSTITUTION

- Letters are often used to represent numbers for general calculations. For example, if you borrow \$1000 (Principal = P) at 12% interest (rate = r) for 2 years (time = t), you could calculate the amount owed at the end of two years by using the general formula below.

$$\begin{aligned}
 \text{Amount owed} &= \text{Principal} + \text{Principal} \times \text{rate} \times \text{time} \\
 &= P + Prt \\
 &= (1000) + (1000)(0.12)(2) \\
 &= 1000 + 240 \\
 &= 1240
 \end{aligned}$$

Amount owed is \$1240

- Note: 1. When substituting, always store the number in a bracket.
 2. Change % to a decimal

EXAMPLES USING SUBSTITUTION

- In the following examples, numbers are substituted (with brackets). Then the order of operation (B.E.M.A.) is completed.

e.g. Evaluate $2xy - 3x^2y$,

if $x = -1$ and $y = 4$

$$\begin{aligned}
 &2xy - 3x^2y \\
 &= 2(-1)(4) - 3(-1)^2(4) \\
 &= 2(-1)(4) - 3(+1)(4) \\
 &= -8 - 12 \\
 &= -20
 \end{aligned}$$

e.g. Evaluate the surface area (A) of a ball,

if $A = 4\pi r^2$ when $r = 5$ and $\pi \approx 3.14$

$$\begin{aligned}
 A &= 4\pi r^2 \\
 &= 4(3.14)(5)^2 \\
 &= 4(3.14)(25) \\
 &= 314
 \end{aligned}$$

EXERCISE III

1. Evaluate the following:

a) $(-2)(-1) + (3)(-2)$

b) $\frac{(-1)(-2)(3)}{-1} - (-2)(-3)$

c) $\frac{(-5)(+6)}{+2} - \frac{3}{4}(-8)$

d) $(5 + 4 - 2)(3) - 2(2+5)$

e) $\frac{(2-5)(-1)}{3} - \frac{(4+6)}{5}$

f) $(-1)^2(2)^3 + (-1)^3(3)^2$

g) $\frac{(-2)^3(-3)^2}{8} + \frac{3+2^2}{2^2}$

h) $3(-1)^2 - 5(5+2-9)^2 - (-2)^3$

2. Substitute and evaluate the following:

a) Amount owed = $P + Prt$, if $P = \$2500$, $r = 9\%$, and $t = 3$ years

b) Amount owed if $P = \$5000$, $r = 11\%$, and $t = \frac{1}{2}$ year

c) $xy - xz$, if $x = 2$, $y = -1$, and $z = -3$

ANSWERS

I Answers (Formula Solving)

1. a) $\frac{A}{l} = w$ b) $V = IR$ c) $\frac{W}{E} = I$ d) $\frac{I}{pr} = t$ e) $\frac{E}{c^2} = m$
- f) $A = \pi r^2$ g) $\frac{3V}{4r^3} = \pi$ h) $r = \frac{mv^2}{F}$ i) $\frac{3V}{\pi r^2} = h$ j) $x = \frac{2a}{c}$
- k) $\frac{2A}{h} = b$ l) $d_2 = \frac{d_1 w_2}{w_1}$ m) $\frac{Fr^2}{m_1 m_2} = k$ n) $e = \frac{rE}{R+r}$
2. a) $\frac{2A}{a+b} = h$ b) $\frac{2A}{h} - c = d$ c) $\frac{9}{5}C + 32 = F$ d) $\frac{2s}{V_1 + V_2} = t$ e) $\frac{2s}{t} - V_1 = V_2$
- f) $sm(t_1 - t_2) = H$

II Answers (Working with Positive and Negative Numbers)

1. 1 2. -4 3. 1 4. -12 5. 13 6. 4 7. 16
8. -24 9. -68 10. -84 11. -40 12. 60 13. -7 14. 6
15. -7 16. 3 17. 96 18. -5 19. -113 20. -9 21. 104
22. 24 23. 49 24. -57 25. -6 26. 4 27. $\frac{1}{4}$ 28. $-1\frac{19}{40}$
29. 7.06 30. $-\frac{1}{6}$ 31. $13\frac{5}{14}$ 32. -6.8138 33. -4.65 34. -7.3 35. $2\overline{18}$
36. $-\frac{5}{8}$ 37. -188.76 38. 4.68 39. $-11\frac{1}{4}$ 40. 13.12 41. $-4\frac{7}{15}$ 42. $\frac{2}{3}$
43. -9.9684 44. $-6\frac{5}{24}$ 45. $-2\frac{1}{40}$ 46. $3\frac{3}{4}$ 47. -10.32 48. 2.8782 49. $6\frac{23}{56}$
50. $-6\frac{27}{65}$ 51. $\frac{14}{33}$ 52. 4 53. $-\frac{13}{20}$ 54. 13

III Answers (Substitution)

1. a) -4 b) -12 c) -9 d) 7 e) -1 f) -1 g) $-7\frac{1}{4}$ h) -9
2. a) \$ 3175.00 b) \$ 5275.00 c) +4

SCIENTIFIC NOTATION

SCIENTIFIC NOTATION WITH 'LARGE' NUMBERS

- A large number can be converted into a one-digit number times the number '10' to a power. This process converts the number into Scientific Notation. Note the list of exponents for the base 10 to the right.

$$\begin{aligned} 10^1 &= 10 \\ 10^2 &= 100 \\ 10^3 &= 1,000 \\ 10^4 &= 10,000 \\ 10^5 &= 100,000 \\ 10^6 &= 1,000,000 \end{aligned}$$

e.g. $2,000,000 = 2 \times 1,000,000$
 $= 2 \times 10^6$

e.g. $3,500 = 3.5 \times 1,000$
 $= 3.5 \times 10^3$

SCIENTIFIC NOTATION WITH 'SMALL' NUMBERS

- A 'small' number (i.e. a decimal less than '1') can also be converted to a one-digit number times the number '10' to a negative power. Note the list of negative exponents for the base 10 to the right.

$$\begin{aligned} 10^{-1} &= \frac{1}{10^1} = .1 \\ 10^{-2} &= \frac{1}{10^2} = .01 \\ 10^{-3} &= \frac{1}{10^3} = .001 \end{aligned}$$

e.g. $.0003 = 3 \times .0001$
 $= 3 \times 10^{-4}$

e.g. $.045 = 4.5 \times .01$
 $= 4.5 \times 10^{-2}$

CONVERTING TO AND FROM SCIENTIFIC NOTATION

- When converting a number into scientific notation, move the decimal place to create a one-digit number. Multiply this one-digit number by the appropriate power of 10.

e.g. $5200 = 5.2 \times 10^3$

e.g. $30,000,000 = 3.0 \times 10^7$

e.g. $.0049 = 4.9 \times 10^{-3}$

e.g. $.000671 = 6.71 \times 10^{-4}$

- When converting from scientific notation into a number, move the decimal point the appropriate number of places to the right (for large numbers) or to the left (for small numbers).

e.g. $3.11 \times 10^3 = 3110$

e.g. $2.5 \times 10^8 = 250,000,000$

e.g. $2.3 \times 10^{-4} = .00023$

e.g. $5.0 \times 10^{-1} = .5$

USING SCIENTIFIC NOTATION

- When using scientific notation, your answer must have a one-digit number. In the following examples, numbers are converted into 'proper' scientific notation (i.e. with a one-digit number).

e.g. $52 \times 10^3 = \frac{5.2}{10^1} \times 10^3$
 $= 5.2 \times 10^4$

e.g. $.15 \times 10^{-4} = \frac{1.5}{10^1} \times 10^{-4}$
 $= 1.5 \times 10^{-5}$

SCIENTIFIC NOTATION AND THE THREE RULES OF EXPONENTS

When using scientific notation, note the following:

- Answers must be in 'proper' scientific notation (i.e. a one-digit number).
- Negative powers of 10 are not converted into positive powers.
- Fractions must be converted to decimals.

RULE I: When multiplying with the same base, add powers.

$$\text{e.g. } (3.0 \times 10^4)(2.0 \times 10^{-7}) = 6.0 \times 10^{-3}$$

$$\begin{aligned} \text{e.g. } 5.1 \times 10^4 \times 3.0 \times 10^7 &= 15.3 \times 10^{11} \\ &= 1.53 \times 10^{12} \end{aligned}$$

RULE II: When dividing with the same base, subtract powers.

$$\text{e.g. } \frac{9 \times 10^{-5}}{2 \times 10^{-1}} = 4.5 \times 10^{-4}$$

$$\begin{aligned} \text{e.g. } \frac{5.0 \times 10^{-4}}{8.0 \times 10^{-7}} &= .625 \times 10^3 \\ &= 6.25 \times 10^2 \end{aligned}$$

RULE III: When a bracket is taken to a power, multiply powers.

$$\begin{aligned} \text{e.g. } (2.0 \times 10^3)^2 &= 2.0^2 \times 10^6 \\ &= 4.0 \times 10^6 \end{aligned}$$

$$\begin{aligned} \text{e.g. } (5 \times 10^{-4})^3 &= 5^3 \times 10^{-12} \\ &= 125 \times 10^{-12} \\ &= 1.25 \times 10^{-10} \end{aligned}$$

EXERCISE

1. Convert to Scientific Notation:

$$\begin{array}{cccccc} \text{a) } 6000 & \text{b) } 250,000 & \text{c) } 36,700,000 & \text{d) } .002 & \text{e) } .00061 & \text{f) } .000005 \end{array}$$

2. Convert to numbers:

$$\begin{array}{ccc} \text{a) } 3.6 \times 10^4 & \text{b) } 5 \times 10^6 & \text{c) } 1.23 \times 10^3 \\ \text{d) } 3 \times 10^{-3} & \text{e) } 2.68 \times 10^{-4} & \text{f) } 1.0 \times 10^{-1} \end{array}$$

3. Convert to 'proper' scientific notation (i.e. a one-digit number):

$$\text{a) } 55 \times 10^4 \quad \text{b) } 23 \times 10^{-3} \quad \text{c) } .5 \times 10^6 \quad \text{d) } .23 \times 10^{-5}$$

4. Simplify the following and write your answer in 'proper' scientific notation:

$$\text{a) } (1.5 \times 10^4)(3.0 \times 10^{-11}) \quad \text{b) } (5.0 \times 10^4)(3.0 \times 10^5)$$

$$\text{c) } (6.2 \times 10^{-3})(2.0 \times 10^{-2}) \quad \text{d) } \frac{8.0 \times 10^6}{4.0 \times 10^{-2}} \quad \text{e) } \frac{1.0 \times 10^{-3}}{2.0 \times 10^{-6}}$$

$$\text{f) } \frac{3.0 \times 10^{-1}}{9.0 \times 10^5} \quad \text{g) } (2 \times 10^4)^3 \quad \text{h) } (5.0 \times 10^{-3})^2$$

$$\text{i) } (5.0 \times 10^{-3})^{-2}$$

ANSWERS

1. a) 6×10^3 b) 2.5×10^5 c) 3.67×10^7
 d) 2×10^{-3} e) 6.1×10^{-4} f) 5×10^{-6}
2. a) 36,000 b) 5 000 000 c) 1 230
 d) .003 e) .000268 f) 0.1
3. a) 5.5×10^5 b) 2.3×10^{-2} c) 5×10^5 d) 2.3×10^{-6}
4. a) 4.5×10^{-7} b) 1.5×10^{10} c) 1.24×10^{-4} d) 2×10^8 e) 5×10^2
 f) $3.\overline{33} \times 10^{-7}$ g) 8×10^{12} h) 2.5×10^{-5} i) 4.0×10^4