

# ABE

## ADULT BASIC EDUCATION

### Math Study Package

This material is intended as a review of skills you have previously learned and which you wish to brush up on before your assessment. We are happy to help you with any part of these review materials that may raise difficulties for you. Please use the **free** help available in the Learning Centre at the campus near you.

**Nanaimo ABE Learning Centre:** Bldg 205 Rm 204.  
No booking is required. Phone (250)740-6425 for hours.

**Cowichan:** Schedule an appointment for an assessment preparation session at Cowichan Campus Learning Centre, Bldg 601, Rm 131. Call 250-746-3509 to book and confirm hours of operation.

**Parksville:** At the Parksville Centre, we *lend* study packages for ABE, and *sell* study packages for all other programs. No Learning Centre.

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## Whole Numbers Review

*Before studying fractions, you should be familiar with all operations on whole numbers. This work will provide a quick review. Answers are at the end on p.4.*

### 1. ADDITION

In addition, remember to begin with the right hand column, and work to the left.

e.g. 
$$\begin{array}{r} 423 \\ + 134 \\ \hline 557 \end{array}$$

Try these:

(a) 
$$\begin{array}{r} 19 \\ + 2 \\ \hline \end{array}$$

(b) 
$$\begin{array}{r} 345 \\ + 123 \\ \hline \end{array}$$

(c) 
$$\begin{array}{r} 951 \\ + 111 \\ \hline \end{array}$$

Remember that when the total of any column is greater than 9, you must carry the left digit in that total to the column to the left.

e.g. 
$$\begin{array}{r} 1 \\ 437 \\ + 126 \\ \hline 563 \end{array}$$

Here is some practice:

(d) 
$$\begin{array}{r} 489 \\ + 96 \\ \hline \end{array}$$

(e) 
$$\begin{array}{r} 2754 \\ + 1666 \\ \hline \end{array}$$

(f) 
$$\begin{array}{r} 238 \\ + 777 \\ \hline \end{array}$$

(g) 
$$\begin{array}{r} 1069 \\ + 888 \\ \hline \end{array}$$

### 2. SUBTRACTION

Remember to begin at the right hand column and move column by column to the left.

e.g. 
$$\begin{array}{r} 768 \\ - 15 \\ \hline 753 \end{array}$$

Do these:

(a) 
$$\begin{array}{r} 1349 \\ - 36 \\ \hline \end{array}$$

(b) 
$$\begin{array}{r} 864 \\ - 333 \\ \hline \end{array}$$

(c) 
$$\begin{array}{r} 3834 \\ - 2222 \\ \hline \end{array}$$

Remember that you should check the accuracy of your answer by adding it to what you took away. If it is correct, your result should be the number you began with. In the example above,  $753 + 15 = 768$ , so we can assume that it is correct.

Remember that when a digit in the bottom number is too big to subtract from the digit in the top number, you borrow from the next column in the top number first.

e.g. 
$$\begin{array}{r} 31 \\ \cancel{4}3 \\ -19 \\ \hline 24 \end{array}$$

You can not subtract 9 from 3, so you borrow 1 ten from the tens column first.

1. Cross out the 4 in the tens place, and replace it with 3.
2. Borrow 10 from 4 to make 13 in the ones column.
3. Subtract 9 from 13
4. Subtract 1 from 3.

Try these:

(d) 
$$\begin{array}{r} 753 \\ -409 \\ \hline \end{array}$$

(e) 
$$\begin{array}{r} 70 \\ -18 \\ \hline \end{array}$$

(f) 
$$\begin{array}{r} 8104 \\ -1987 \\ \hline \end{array}$$

(g) 
$$\begin{array}{r} 7512 \\ -943 \\ \hline \end{array}$$

(h) 
$$\begin{array}{r} 1000 \\ -369 \\ \hline \end{array}$$

(i) 
$$\begin{array}{r} 3084 \\ -2294 \\ \hline \end{array}$$

### 3. MULTIPLICATION

Take the time to learn your multiplication tables. That will save you a lot of time later on!

Multiply each digit in the top number by the bottom number.

Write the answer from right to left, starting in the ones column.

e.g. 
$$\begin{array}{r} 12 \\ \times 4 \\ \hline 48 \end{array}$$

$$\begin{array}{r} 41 \\ \times 9 \\ \hline 369 \end{array}$$

Remember that any number multiplied by 0 is 0.

Try these:

(a) 
$$\begin{array}{r} 430 \\ \times 3 \\ \hline \end{array}$$

(b) 
$$\begin{array}{r} 702 \\ \times 41 \\ \hline \end{array}$$

(c) 
$$\begin{array}{r} 3011 \\ \times 7 \\ \hline \end{array}$$

When you multiply by a two or three digit number, be sure to begin your answer under the ones column.

Then continue on the line below the tens and then the hundreds. Always multiply from right to left.

Leave a space under the bottom digit that has already been multiplied OR add a zero in that space to keep the columns lined up neatly and correctly.

Carrying in multiplication is like carrying in addition. Multiply first and then add the number being carried. Line up the digits carefully under the correct column.

e.g. 
$$\begin{array}{r} 23 \\ \times 22 \\ \hline 46 \\ 46 \\ \hline 506 \end{array}$$

$$\begin{array}{r} 638 \\ \times 51 \\ \hline 638 \\ 3190 \\ \hline 32538 \end{array}$$

$$\begin{array}{r} 405 \\ \times 266 \\ \hline 2430 \\ 24300 \\ 81000 \\ \hline 107730 \end{array}$$

Now try these:

$$\begin{array}{r} \text{(d)} \quad 789 \\ \times 46 \\ \hline \end{array}$$

$$\begin{array}{r} \text{(e)} \quad 5106 \\ \times 203 \\ \hline \end{array}$$

When you multiply by 10, 100, 1000, etc., add the same number of zeroes to the right of the number.

e.g.  $35 \times 10 = 350$

$41 \times 100 = 4100$

$60 \times 1000 = 60\,000$       Tricky!

#### 4. DIVISION

You will need your multiplication tables here too! Learn them well!

Division is the opposite of multiplication. That helps you to check your division answer.

Pay careful attention to lining up digits, as you did in multiplying large numbers, so that you can keep your working straight.

Here are the words: 168 (the **dividend**) divided by 7 (the **divisor**) is 24 (the **quotient**) **OR** 7 into 168 is 24.

$$\begin{array}{r} \text{e.g.} \quad 24 \\ 7 \overline{)168} \\ \underline{-14} \phantom{0} \\ 28 \\ \underline{-28} \\ 0 \end{array}$$

Step 1: Divide the divisor 7 into 16 = 2. Place the 2 above the 6.

Step 2: Multiply  $7 \times 2 = 14$ . Place the 14 under the 16.

Step 3: Subtract:  $16 - 14 = 2$ .

Step 4: Bring down the next number to the right = 8

Step 5: The new number is 28.

Step 6: Divide 7 into 28 = 4. Multiply 4 by the divisor 7 and the answer is 28.

Step 7: Subtract 28 from 28 = 0. The division is complete.

**Check:** In division, it is easy to check the answer by multiplying your answer (the **quotient**) by the number you are dividing by (the **divisor**).  
 $24 \times 7 = 168$  so the answer is likely to be correct.

Sometimes there is an amount left over, which is called the **remainder**.

It is placed on the top line with the letter **r** for **remainder** and is part of the **quotient**.

$$\begin{array}{r} \text{e.g.} \quad 569 \text{ r } 3 \\ 6 \overline{)3417} \\ \underline{-30} \phantom{0} \\ 41 \\ \underline{-36} \\ 57 \\ \underline{-54} \\ 3 \end{array}$$

Try these:

(a)  $4 \overline{)1631}$

(b)  $7 \overline{)1046}$

(c)  $9 \overline{)3004}$

When you check your answer for a division problem with a remainder, multiply your answer (the **quotient**) by the number you are dividing by (the **divisor**) and **add the remainder**.

When you divide by a double or triple digit number, you need to use another different skill. You need to **estimate**, which is a process of thoughtful guessing. This takes time and practice.

$$\begin{array}{r} 223 \\ 28 \overline{)6244} \\ \underline{-56} \\ 64 \\ \underline{-56} \\ 84 \\ \underline{-84} \\ 0 \end{array}$$

Step 1: Think of *about* how many times 28 goes into 6244.

To do that, **round** 28 to 30 ( $28 \approx 30$ ) and divide that into 62.

The answer is **approximately or close to 2**.

Step 2: Place 2 above the last digit of 62.

Step 3:  $2 \times 28$  (the divisor) = 56. Then  $62 - 56 = 6$ .

Step 4: Bring down 4. **Estimate** how many times 28,  $\approx 30$ , divides into  $64 \approx 2$ . Place 2 on the top line.

Step 5:  $2 \times 28 = 56$ .  $64 - 56 = 8$ .

Step 6: Bring down 4. **Estimate** how many times 28,  $\approx 30$ , divides into  $84 \approx 3$ .

Sometimes your **estimate** is not quite right, but it should only be 1 more or 1 less than the correct number, so the **estimate** gives a good starting point. Here is an example with a **remainder**:

$$\begin{array}{r} 337 \text{ r } 32 \\ 41 \overline{)13849} \\ \underline{-123} \\ 154 \\ \underline{-123} \\ 319 \\ \underline{-287} \\ 32 \end{array}$$

Step 1: **Round**  $41 \approx 40$ . Divide 40 into the first part of the number.

Step 2: 40 goes into 138 about 3 times. Place 3 above the last digit of 138.

Step 3:  $3 \times 41 = 123$ . Subtract 123 from 138 = 15.

Step 4: Bring down 4. Estimate how many times  $41 \approx 40$  goes into  $154 \approx 3$ .

Step 5: Multiply  $3 \times 41 = 123$ . 123 from 154 = 31.

Step 6: Bring down 9. Estimate how many times  $41 \approx 40$  goes into  $319 \approx 7$ .  $7 \times 41 = 287$ .

Step 7:  $319 - 287 = 32$ . Divide 41 into 32. It cannot divide, so 32 is the remainder.

Try these:

$$(d) \quad 57 \overline{)3477}$$

$$(e) \quad 36 \overline{)7601}$$

$$(f) \quad 67 \overline{)13467}$$

$$(g) \quad 29 \overline{)13796}$$

### ANSWERS

<b>Addition:</b>	(a) 21	(b) 468	(c) 1062	(d) 585	(e) 4420	(f) 1015	(g) 1957
<b>Subtraction:</b>	(a) 1313	(b) 531	(c) 1612	(d) 344	(e) 52	(f) 6117	(g) 6569
	(h) 631	(i) 790					
<b>Multiplication:</b>	(a) 1290	(b) 28782	(c) 21077	(d) 36294	(e) 1036518		
<b>Division:</b>	(a) 407 r 3	(b) 149 r 3	(c) 333 r 7	(d) 61	(e) 211 r 5	(f) 201	(g) 475 r 21

# FRACTIONS

**This material is intended as a review of skills you once learned and wish to review before your assessment.**

Before studying Algebra, you should be familiar with all of the topics included here. Read the explanations carefully, and do as many of each type of question in each exercise until you have mastered the material.

Check your answers in the answer key at the end.

**Do not** use a calculator, unless you are instructed to.

***ANSWER KEY AT END***

## FRACTION REVIEW

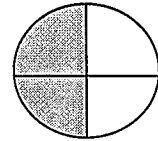
### A. INTRODUCTION

#### 1. What is a fraction?

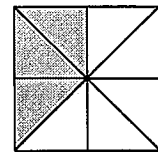
A fraction consists of a numerator (part) on top of a denominator (total) separated by a horizontal line.  
For example, the fraction of the circle which is shaded is:

**2 (parts shaded)**  
**4 (total parts)**

Fraction =  $\frac{\text{numerator}}{\text{denominator}}$

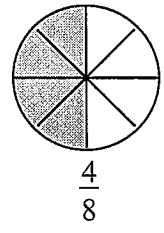
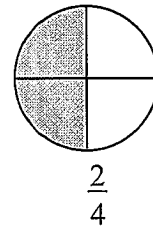
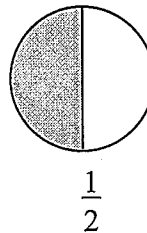


In the square on the right, the fraction shaded is  $\frac{3}{8}$  and  
the fraction unshaded is  $\frac{5}{8}$



#### 2. Equivalent Fractions – Multiplying

The three circles on the right each have equal parts shaded, yet are represented by different but equal fractions. These fractions, because they are equal, are called equivalent fractions.



*Any fraction can be changed into an equivalent fraction*

by multiplying **both** the numerator and denominator by the same number

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4} \quad \text{or} \quad \frac{1 \times 4}{2 \times 4} = \frac{4}{8} \quad \text{so} \quad \frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Similarly

$$\frac{5 \times 2}{9 \times 2} = \frac{10}{18} \quad \text{or} \quad \frac{5 \times 3}{9 \times 3} = \frac{15}{27} \quad \text{so} \quad \frac{5}{9} = \frac{10}{18} = \frac{15}{27}$$

You can see from the above examples that each fraction has an infinite number of fractions that are equivalent to it.

### 3. Equivalent Fractions – Dividing (Reducing)

Equivalent fractions can also be created if both the numerator and denominator can be divided by the same number (a factor) evenly.

This process is called “reducing a fraction” by dividing a common factor (a number which divides into both the numerator and denominator evenly).

$$\frac{4}{8} \div \frac{4}{4} = \frac{1}{2}$$

$$\frac{27}{81} \div \frac{9}{9} = \frac{3}{9}$$

$$\frac{5}{30} \div \frac{5}{5} = \frac{1}{6}$$

$$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

### 4. Simplifying a Fraction (Reducing to its Lowest Terms)

It is usual to reduce a fraction until it can't be reduced any further.

A simplified fraction has no common factors which will divide into both numerator and denominator.

Notice that, since 27 and 81 have a common factor of 9, we find that  $\frac{3}{9}$  is an equivalent fraction.

But this fraction has a factor of 3 common to both numerator and denominator.

So, we must reduce this fraction again. It is difficult to see, but if we had known that 27 was a factor (divides into both parts of the fraction evenly), we could have arrived at the answer in one step

e.g.  $\frac{8}{24} \div \frac{8}{8} = \frac{1}{3}$        $\frac{45}{60} \div \frac{15}{15} = \frac{3}{4}$

$$\frac{27}{81} \div \frac{9}{9} = \frac{3}{9}$$

$$\frac{3}{9} \div \frac{3}{3} = \frac{1}{3}$$

$$\frac{27}{81} \div \frac{27}{27} = \frac{1}{3}$$

5. EXERCISE 1: Introduction to Fractions

a) Find the missing part of these equivalent fractions

1)  $\frac{2}{3} = \frac{\quad}{6}$

2)  $\frac{3}{4} = \frac{\quad}{12}$

3)  $\frac{5}{8} = \frac{\quad}{40}$

4)  $\frac{1}{16} = \frac{\quad}{32}$

5)  $\frac{2}{15} = \frac{\quad}{45}$

6.)  $\frac{7}{9} = \frac{\quad}{27}$

7)  $\frac{7}{10} = \frac{\quad}{100}$

8)  $\frac{3}{4} = \frac{\quad}{44}$

$\times 2$
<u>Example:</u> $\frac{3}{5} = \frac{\quad}{10}$
$\times 2$
Since $5 \times 2 = 10$ , multiply the numerator by 2, also.
So, $\frac{3}{5} = \frac{6}{10}$

b) Find the missing part of these equivalent fractions.

1)  $\frac{8}{16} = \frac{\quad}{4}$

2)  $\frac{24}{27} = \frac{\quad}{9}$

3)  $\frac{6}{10} = \frac{\quad}{5}$

4)  $\frac{25}{35} = \frac{\quad}{7}$

5)  $\frac{20}{30} = \frac{\quad}{6}$

6)  $\frac{90}{100} = \frac{\quad}{50}$

$\div 5$
<u>Example:</u> $\frac{5}{10} = \frac{\quad}{2}$
$\div 5$
Since $10 \div 5 = 2$ divide the numerator by 5, also.
So, $\frac{5}{10} = \frac{1}{2}$

c) Simplify the following fractions (reduce to lowest terms).

1)  $\frac{9}{12}$

2)  $\frac{8}{12}$

3)  $\frac{6}{8}$

4)  $\frac{15}{20}$

5)  $\frac{20}{25}$

6)  $\frac{14}{21}$

7)  $\frac{8}{16}$

8)  $\frac{24}{36}$

9)  $\frac{66}{99}$

10)  $\frac{18}{30}$

## B. TYPES OF FRACTIONS

### 1. Common Fractions

A common fraction is one in which the numerator is less than the denominator (or a fraction which is less than the number 1). A common fraction can also be called a proper fraction.

e.g.  $\frac{1}{2}$ ,  $\frac{3}{4}$ ,  $\frac{88}{93}$ ,  $\frac{8}{15}$  are all common fractions.

### 2. Fractions that are Whole Numbers

Some fractions, when reduced, are really whole numbers (1, 2, 3, 4... etc). Whole numbers occur if the denominator divides into the numerator evenly.

e.g.  $\frac{8}{4}$  is the same as  $\frac{8}{4} \div \frac{4}{4} = \frac{2}{1}$  or 2  
 $\frac{30}{5}$  is the same as  $\frac{30}{5} \div \frac{5}{5} = \frac{6}{1}$  or 6

So, the fraction  $\frac{30}{5}$  is really the whole number 6.

Notice that a whole number can always be written as a fraction with a denominator of 1.

e.g.  $10 = \frac{10}{1}$

### 3. Mixed Numbers

A mixed number is a combination of a whole number and a common fraction.

e.g.  $2\frac{3}{5}$  (two and three-fifths)  
 $27\frac{2}{9}$  (twenty-seven and two-ninths)  
 $9\frac{3}{6} = 9\frac{1}{2}$  (always reduce fractions)

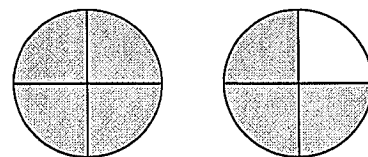
#### 4. Improper Fractions

An improper fraction is one in which the numerator is larger than the denominator.

From the circles on the right, we see that  $1\frac{3}{4}$  (mixed number)

is the same as  $\frac{7}{4}$  (improper fraction).

An improper fraction, like  $\frac{7}{4}$ , can be changed to a mixed number by dividing the denominator into the numerator and expressing the remainder (3) as the numerator.



$$1\frac{3}{4} = \frac{7}{4}$$

$$\frac{7}{4} = 4 \overline{)7} = 1\frac{3}{4}$$

e.g.  $\frac{16}{5} = 3\frac{1}{5}$        $\frac{29}{8} = 3\frac{5}{8}$        $\frac{14}{3} = 4\frac{2}{3}$

A mixed number can be changed to an improper fraction by changing the whole number to a fraction with the same denominator as the common fraction.

$$2\frac{3}{5} = \frac{10}{5} + \frac{3}{5} = \frac{13}{5}$$

$$10\frac{1}{9} = \frac{90}{9} + \frac{1}{9} = \frac{91}{9}$$

A simple way to do this is to multiply the whole number by the denominator, and then add the numerator.

e.g.  $4\frac{5}{9} = \frac{4 \times 9 + 5}{9} = \frac{36 + 5}{9} = \frac{41}{9}$

$$10\frac{2}{7} = \frac{10 \times 7 + 2}{7} = \frac{70 + 2}{7} = \frac{72}{7}$$

#### 5. Simplifying fractions

All types of fractions must always be simplified (reduced to lowest terms).

e.g.  $\frac{6}{9} = \frac{2}{3}$ ,       $2\frac{5}{25} = 2\frac{1}{5}$ ,       $\frac{27}{18} = \frac{3}{2} = 1\frac{1}{2}$

Note that many fractions can not be reduced since they have no common factors.

e.g.  $\frac{17}{21}$ ,  $\frac{4}{9}$ ,  $\frac{18}{37}$

6. EXERCISE 2 : Types of Fractions

a) Which of the following are common fractions (C), whole numbers (W), mixed numbers (M) or improper fractions (I)?

1)  $\frac{2}{3}$

2)  $3\frac{4}{5}$

3)  $\frac{7}{5}$

4)  $\frac{8}{8}$

5)  $\frac{24}{2}$

6)  $5\frac{8}{19}$

7)  $2\frac{3}{3}$

8)  $\frac{25}{24}$

9)  $\frac{24}{25}$

10)  $\frac{12}{12}$

b) Change the following to mixed numbers:

1)  $\frac{7}{5}$

2)  $\frac{18}{11}$

3)  $\frac{70}{61}$

4)  $\frac{12}{5}$

5)  $\frac{100}{99}$

6)  $\frac{25}{2}$

c) Change the following to improper fractions:

1)  $2\frac{1}{5}$

2)  $6\frac{3}{8}$

3)  $8\frac{2}{3}$

4)  $11\frac{1}{5}$

5)  $9\frac{4}{5}$

6)  $4\frac{3}{4}$

d) Simplify the following fractions:

1)  $\frac{28}{40}$

2)  $\frac{80}{10}$

3)  $2\frac{12}{18}$

4)  $5\frac{27}{54}$

5)  $\frac{25}{15}$

6)  $\frac{90}{12}$

### C. COMPARING FRACTIONS

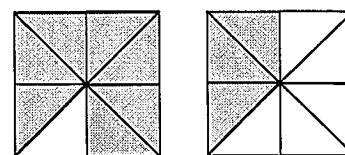
In the diagram on the right, it is easy to see that  $\frac{7}{8}$  is larger than  $\frac{3}{8}$  (since 7 is larger than 3).

However, it is not as easy to tell that  $\frac{7}{8}$  is larger than  $\frac{5}{6}$ .

In order to compare fractions, we must have the same (common) denominators. This process is called

“Finding the Least Common Denominator” and is usually

abbreviated as finding the LCD or LCM (lowest common multiple).



$$\frac{7}{8} = \frac{3}{8}$$

Which is larger:  $\frac{7}{8}$  or  $\frac{5}{6}$ ?

In order to compare these fractions, we must change both fractions to equivalent fractions with a common denominator.

To do this, take the largest denominator (8) and examine multiples of it, until the other denominator (6) divides into it.

Notice that, when we multiply  $8 \times 3$ , we get 24, which 6 divides into.

Now change the fractions to 24<sup>th</sup> s.

When we change these fractions to equivalent fractions with an LCD of 24, we can easily see

that  $\frac{7}{8}$  is larger than  $\frac{5}{6}$  since  $\frac{21}{24}$  is greater than  $\frac{20}{24}$ .

$$8 \times 1 = 8$$

(6 doesn't divide into 8)

$$8 \times 2 = 16$$

(6 doesn't divide into 16)

$$8 \times 3 = 24 \quad \text{LCD}$$

$$\begin{array}{r} \times 3 \\ \times 3 \end{array}$$

$$\begin{array}{r} \times 4 \\ \times 4 \end{array}$$

$$\frac{7}{8} = \frac{21}{24} \quad \frac{5}{6} = \frac{20}{24}$$

Which is larger:  $\frac{4}{9}$  or  $\frac{5}{12}$ ?

Examine multiples of the larger denominator (12) until the smaller denominator divides into it. This tells us that the LCD is 36.

Now, we change each fraction to equivalent fractions with the LCD of 36.

$$12 \times 1 = 12$$

$$12 \times 2 = 24$$

$$12 \times 3 = 36 \text{ (LCD)}$$

$$\text{LCD is 36.}$$

$$\begin{array}{r} \text{x4} \\ \frac{4}{9} = \frac{16}{36} \\ \text{x4} \end{array}$$

$$\begin{array}{r} \text{x3} \\ \frac{5}{12} = \frac{15}{36} \\ \text{x3} \end{array}$$

So,  $\frac{4}{9}$  is larger than  $\frac{5}{12}$

Which is larger:  $\frac{4}{5}$  or  $\frac{13}{15}$  or  $\frac{11}{12}$ ?

Find the LCD by examining multiples of 15. Notice that, when we multiply  $15 \times 4$ , we find that 60 is the number that all denominators divide into.

$$15 \times 1 = 15$$

$$15 \times 2 = 30$$

$$15 \times 3 = 45$$

$$15 \times 4 = 60 \text{ (LCD)}$$

$$\begin{array}{r} \text{x12} \\ \frac{4}{5} = \frac{48}{60} \\ \text{x12} \end{array}$$

$$\begin{array}{r} \text{x4} \\ \frac{13}{15} = \frac{52}{60} \\ \text{x4} \end{array}$$

$$\begin{array}{r} \text{x5} \\ \frac{11}{12} = \frac{55}{60} \\ \text{x5} \end{array}$$

So,  $\frac{11}{12}$  is the largest fraction.

Which is larger:  $\frac{7}{9}$  or  $\frac{13}{18}$ ?

Notice that one denominator (9) divides into the other denominator (18). This means that the LCD = 18 and we only have to change one fraction  $\left(\frac{7}{9}\right)$  to an equivalent fraction.

$$\begin{array}{r} \text{x2} \\ \frac{7}{9} = \frac{14}{18} \\ \text{x2} \end{array}$$

So,  $\frac{7}{9}$  is larger than  $\frac{13}{18}$

1. **EXERCISE 3: Comparing Fractions**

Which is the largest fraction? (find LCD first)

1)  $\frac{7}{13}$  or  $\frac{6}{13}$

2)  $1\frac{9}{10}$  or  $\frac{18}{10}$

3)  $\frac{4}{5}$  or  $\frac{9}{10}$

4)  $\frac{3}{13}$  or  $\frac{5}{12}$

5)  $\frac{5}{8}$  or  $\frac{4}{7}$

6)  $\frac{1}{2}$  or  $\frac{6}{11}$  or  $\frac{7}{12}$

7)  $\frac{2}{3}$  or  $\frac{11}{15}$

8)  $\frac{4}{9}$  or  $\frac{5}{12}$  or  $\frac{3}{8}$

9)  $\frac{1}{4}$  or  $\frac{3}{16}$

**D. ADDING FRACTIONS**

There are four main operations that we can do with numbers: addition ( + ), subtraction ( - ), multiplication ( x ), and division ( ÷ ).

In order to add or subtract, fractions must have common denominators.

This is not required for multiplication or division.

**1. Adding with Common Denominators**

To add fractions, if the denominators are the same, we simply add the numerators and keep the same denominators.

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

e.g.

Add $\frac{1}{12}$ and $\frac{5}{12}$
---------------------------------------

Since the denominators are common, simply add the numerators. Notice that we must reduce the answer, if possible.

$$\begin{aligned} \frac{1}{12} + \frac{5}{12} &= \frac{6}{12} \\ &= \frac{1}{2} \end{aligned}$$

**2. Adding When One Denominator is a Multiple of the Other**

Add $\frac{2}{9}$ and $\frac{5}{27}$
--------------------------------------

$$\begin{aligned} \frac{2}{9} &= \frac{6}{27} \\ &\quad \times 3 \\ \frac{5}{27} &\quad \times 3 \end{aligned}$$

Notice that the denominators are not common. Also notice that 27 is a multiple of 9 (since  $9 \times 3 = 27$ ). This means that the LCD = 27 (see the last example in “Comparing Fractions”).

$$\begin{aligned} \frac{2}{9} + \frac{5}{27} &= \frac{6}{27} + \frac{5}{27} \\ &= \frac{11}{27} \end{aligned}$$

3. Adding Any Fraction

Add $\frac{7}{12}$ and $\frac{13}{15}$
--

$$\begin{aligned}\frac{7}{12} + \frac{13}{15} &= \frac{35}{60} + \frac{52}{60} \\ &= \frac{87}{60} \\ &= 1\frac{9}{20}\end{aligned}$$

We must find a common denominator by examining multiples of the largest denominator. We find that the LCD = 60.

Add $1\frac{5}{6}$ and $2\frac{3}{8}$
---------------------------------------

When adding mixed numbers, add the whole numbers and the fractions separately. Find common denominators and add.

$$\begin{array}{r} 1\frac{5}{6} = 1\frac{20}{24} \\ + 2\frac{3}{8} = 2\frac{9}{24} \\ \hline \end{array}$$

If an improper fraction occurs in the answer, change it to a common fraction by doing the following.

$$\begin{array}{r} \text{total equals } 3\frac{29}{24} \\ 3\frac{29}{24} = 3 + 1\frac{5}{24} \\ = 4\frac{5}{24} \end{array}$$

4. The Language of Addition

$\frac{1}{2} + \frac{2}{3}$ CAN BE WORDED
---

$\frac{1}{2}$  plus  $\frac{2}{3}$

$\frac{1}{2}$  and  $\frac{2}{3}$

total of  $\frac{1}{2}$  and  $\frac{2}{3}$

sum of  $\frac{1}{2}$  and  $\frac{2}{3}$

addition of  $\frac{1}{2}$  and  $\frac{2}{3}$

$\frac{1}{2}$  combined with  $\frac{2}{3}$

$\frac{1}{2}$  more than (or greater than)  $\frac{2}{3}$

Note: All of these can be worded with the fractions in reverse order:

e.g.  $\frac{2}{3}$  plus  $\frac{1}{2}$  is the same as  $\frac{1}{2}$  plus  $\frac{2}{3}$

5. EXERCISE 4: Adding Fractions

a) Add the following:

1)  $\frac{1}{5} + \frac{2}{5}$

2)  $\frac{4}{5} + \frac{3}{5}$

3)  $\frac{4}{9} + \frac{2}{9}$

4)  $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

b) Find the sum of:

1)  $\frac{2}{3} + \frac{1}{9}$

2)  $\frac{1}{2} + \frac{3}{8}$

3)  $\frac{1}{4} + \frac{5}{16}$

4)  $\frac{2}{3} + \frac{4}{15}$

c) Add the following:

1)  $3\frac{1}{2} + 4\frac{1}{4}$

2)  $9\frac{2}{3} + 3\frac{1}{6}$

3)  $8\frac{1}{2} + 4\frac{4}{5}$

4)  $2\frac{3}{4} + 6\frac{1}{2}$

5)  $4\frac{1}{3} + 6\frac{5}{6}$

6)  $6\frac{1}{3} + 8\frac{3}{4}$

7)  $7\frac{2}{3} + \frac{4}{5}$

8)  $8\frac{2}{3} + 6\frac{1}{4} + 1\frac{3}{8}$

d) Evaluate the following:

1)  $\frac{2}{3}$  and  $\frac{3}{7}$

2) total of  $\frac{5}{6}$  and  $\frac{3}{8}$

3)  $\frac{1}{2}$  plus  $\frac{1}{5}$

4)  $\frac{3}{2}$  greater than  $\frac{5}{7}$

5)  $\frac{5}{12}$  combined with  $\frac{3}{8}$

6) sum of  $\frac{1}{6}$  and  $\frac{3}{14}$

## E. SUBTRACTING FRACTIONS

### 1. Common Fractions

As in addition, we must have common denominators in order to subtract. Find the LCD; change the fractions to equivalent fraction with the LCD as the denominator. Then subtract the numerators, but keep the same denominator.

$$\frac{5}{8} - \frac{3}{8} = \frac{2}{8} \text{ or } \frac{1}{4}$$

$$\begin{aligned} \frac{2}{3} - \frac{3}{8} &= \frac{16}{24} - \frac{9}{24} \\ &= \frac{7}{24} \end{aligned}$$

### 2. Mixed Numbers

When subtracting whole numbers, subtract the whole numbers, and then subtract the fractions separately.

$$3\frac{5}{9} - 1\frac{3}{9} = 2\frac{2}{9}$$

However, if the common fraction we are subtracting is smaller than the other common fraction, we must borrow the number "1" from the large whole number.

i.e.  $4\frac{2}{7} = 3 + \frac{7}{7} + \frac{2}{7}$ , or  $3\frac{9}{7}$

$$\begin{aligned} 4\frac{2}{7} - 2\frac{5}{7} &= 3\frac{9}{7} - 2\frac{5}{7} \\ &= 1\frac{4}{7} \end{aligned}$$

To subtract  $1\frac{3}{4}$  from  $6\frac{2}{3}$ , first change the common fractions to equivalent fractions with the LCD. Since  $\frac{8}{12}$  is smaller than  $\frac{9}{12}$ , borrow from 6.

$$6\frac{8}{12} = 5\frac{12}{12} + \frac{8}{12} = 5\frac{20}{12}$$

$$\begin{aligned} 6\frac{2}{3} - 1\frac{3}{4} &= 6\frac{8}{12} - 1\frac{9}{12} \\ &= 5\frac{20}{12} - 1\frac{9}{12} \\ &= 4\frac{11}{12} \end{aligned}$$

### 3. The Language of Subtraction

$\frac{5}{6} - \frac{2}{3}$ CAN BE WORDED
---

$$\frac{5}{6} \text{ minus } \frac{2}{3} \text{ (NOT } \frac{2}{3} \text{ minus)}$$

$$\frac{2}{3} \text{ subtracted from } \frac{5}{6}$$

$$\frac{2}{3} \text{ from } \frac{5}{6}$$

$$\frac{2}{3} \text{ less than } \frac{5}{6}$$

$$\frac{5}{6} \text{ decreased by or lowered by } \frac{2}{3}$$

$$\text{the difference of } \frac{5}{6} \text{ and } \frac{2}{3}$$

**NOTE:** Unlike addition, we can not reword the above with the fractions in reverse order:

i.e.  $\frac{1}{2} - \frac{2}{3}$  is **NOT** the same as  $\frac{2}{3} - \frac{1}{2}$

### 4. EXERCISE 5: Subtracting Fractions

a) Subtract the following:

1)  $\frac{9}{12} - \frac{1}{8}$

2)  $\frac{14}{15} - \frac{1}{6}$

3)  $\frac{5}{6} - \frac{3}{8}$

4)  $\frac{7}{9} - \frac{2}{3}$

5)  $9\frac{2}{3} - 6\frac{1}{6}$

6)  $4\frac{1}{2} - 1\frac{1}{4}$

7)  $\frac{3}{4} - \frac{5}{8}$

8)  $\frac{11}{12} - \frac{2}{3}$

b) Subtract the following:

1)  $6\frac{1}{3} - 2\frac{2}{3}$

2)  $13\frac{1}{4} - 5\frac{3}{4}$

3)  $5\frac{5}{7} - 4\frac{6}{7}$

4)  $4\frac{3}{4} - 1\frac{11}{12}$

5)  $16\frac{2}{3} - 5\frac{3}{4}$

6)  $9\frac{1}{6} - 4\frac{3}{8}$

c) Find the following:

1) What is  $\frac{5}{8}$  minus  $\frac{3}{16}$ ?

2)  $\frac{2}{7}$  decreased by  $\frac{1}{21}$  is what?

3) What is  $\frac{4}{9}$  less than  $\frac{7}{9}$ ?

4) What is  $\frac{1}{6}$  from  $\frac{9}{24}$ ?

## F. MULTIPLYING FRACTIONS

### 1. Common Fractions

When multiplying fractions, a common denominator is not needed. Simply multiply the numerators and multiply the denominators separately.

$$\frac{2}{3} \times \frac{5}{9} = \frac{2 \times 5}{3 \times 9} = \frac{10}{27}$$

Sometimes, we can reduce the fractions before multiplying.

$$\frac{3}{5} \times \frac{5}{7} = \frac{3 \times \overset{1}{\cancel{5}}}{\underset{1}{\cancel{5}} \times 7} = \frac{3}{7}$$

Any common factor in either numerator can cancel with the same factor in the denominator. Multiply after cancelling (reducing).

$$\frac{4}{9} \times \frac{3}{8} = \frac{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \times \underset{2}{\cancel{8}}} = \frac{1}{6}$$

Note that any whole number (16) has the number "1" understood in its denominator.

$$\frac{5}{8} \times 16 = \frac{5 \times \overset{2}{\cancel{16}}}{\underset{1}{\cancel{8}} \times 1} = 10$$

If more than two fractions are multiplied, the same principles apply.

$$\frac{2}{9} \times \frac{3}{20} \times \frac{1}{4} = \frac{2 \times 3 \times 1}{9 \times 20 \times 4} = \frac{1}{120}$$

### 2. Mixed Numbers

Mixed numbers must be changed to improper fractions before multiplying. Remember that a mixed number (like  $2\frac{3}{4}$ ) can be changed to an improper fraction by multiplying the whole number (2) by the denominator (4) and then adding the numerator.

$$2\frac{3}{4} \times 4\frac{8}{11} = \frac{11}{4} \times \frac{52}{11} = 13$$

See page 6 for instructions on changing a mixed number to an improper fraction.

$$\begin{aligned} 5\frac{4}{9} \times 2\frac{3}{7} &= \frac{49}{9} \times \frac{17}{7} \\ &= \frac{119}{9} \\ &\text{or } 13\frac{2}{9} \end{aligned}$$

3. The Language of Multiplication

$\frac{1}{2} \times \frac{2}{3}$ CAN BE WORDED
--

$$\frac{1}{2} \text{ multiplied by } \frac{2}{3}$$

$$\frac{1}{2} \text{ by } \frac{2}{3}$$

$$\frac{1}{2} \text{ of } \frac{2}{3}$$

$$\text{the product of } \frac{1}{2} \text{ and } \frac{2}{3}$$

NOTE: When multiplying, it doesn't matter which fraction is first.

$$\text{i.e. } \frac{1}{2} \times \frac{2}{3} \text{ is the same as } \frac{2}{3} \times \frac{1}{2}$$

4. EXERCISE 6: Multiplying Fractions

a) Multiply (cancel first, when possible):

1)  $\frac{2}{3} \times \frac{3}{4}$

2)  $\frac{8}{9} \times \frac{12}{16}$

3)  $\frac{5}{3} \times \frac{9}{15}$

4)  $\frac{2}{5} \times \frac{15}{21}$

5)  $\frac{5}{8} \times \frac{48}{125}$

6)  $\frac{3}{4} \times \frac{16}{27} \times \frac{9}{16}$

7)  $\frac{5}{3} \times \frac{7}{4} \times 90 \times 20 \times \frac{16}{14}$

8)  $2\frac{4}{5} \times 3\frac{4}{7}$

9)  $8\frac{2}{3} \times \frac{15}{21}$

10)  $\frac{18}{19} \times \frac{57}{4} \times \frac{6}{32} \times \frac{8}{9} \times \frac{2}{3}$

11)  $1\frac{12}{25} \times 2\frac{1}{37}$

12)  $2\frac{5}{12} \times 6$

b) Find the following:

1) What is  $\frac{2}{3}$  of 45?

2)  $\frac{41}{7}$  by 14 is what number?

3) What is  $\frac{5}{9}$  times  $\frac{27}{10}$ ?

4)  $\frac{4}{5}$  of  $2\frac{1}{7}$  is what number?

5)  $\frac{1}{2}$  of  $\frac{62}{63}$  is what number?

## G. DIVIDING FRACTIONS

To divide fractions, we invert (take the reciprocal of) the fraction that we are dividing by, then cancel (reduce), and then multiply.

Taking the reciprocal of a fraction involves “flipping” the fraction so that the numerator and denominator switch places.

$$\frac{2}{3} \text{ reciprocal } \frac{3}{2}$$

$$\frac{8}{19} \text{ reciprocal } \frac{19}{8}$$

Note that a whole number is really a fraction (e.g.  $4 = \frac{4}{1}$ ).

$$4 \text{ reciprocal } \frac{4}{1}$$

### 1. Common Fractions

Simply invert (take the reciprocal of) the fractions that we are dividing by ( $\frac{8}{9}$ ). Then cancel and multiply.

$$\frac{5}{7} \div \frac{8}{9} = \frac{5}{7} \times \frac{9}{8} = \frac{45}{56}$$

**Note:** you can only cancel **after** the division is changed to a multiplication.

$$\frac{9}{16} \div \frac{3}{32} = \frac{9}{16} \times \frac{32}{3} = 6$$

### 2. Mixed Numbers

As in multiplication, mixed numbers must be changed to improper fractions.

$$\begin{aligned} 1\frac{2}{3} \div 2\frac{1}{7} &= \frac{5}{3} \div \frac{15}{7} \\ &= \frac{5}{3} \times \frac{7}{15} \\ &= \frac{7}{9} \end{aligned}$$

### 3. The Language of Division

$$\frac{1}{2} \div \frac{2}{3} \text{ CAN BE WORDED}$$

$$\frac{1}{2} \text{ divided by } \frac{2}{3}$$

$$\frac{2}{3} \text{ into } \frac{1}{2}$$

$$\text{divide } \frac{1}{2} \text{ by } \frac{2}{3}$$

**NOTE:** In multiplication, the order of the fractions was not important.

i.e.  $\frac{1}{2} \times \frac{2}{3}$  is the same as  $\frac{2}{3} \times \frac{1}{2}$

In division, this is not the case. The order of the fractions is important.  
Consider the following:

$$\frac{1}{2} \div \frac{2}{3} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

$$\text{but } \frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} = 1\frac{1}{3}$$

### 4. EXERCISE 7: Dividing Fractions

a) Divide:

1)  $\frac{3}{5} \div \frac{9}{15}$

2)  $\frac{3}{7} \div \frac{12}{5}$

3)  $6 \div \frac{2}{3}$

4)  $\frac{7}{12} \div 14$

5)  $\frac{13}{15} \div \frac{39}{40}$

6)  $3\frac{3}{10} \div \frac{11}{15}$

7)  $\frac{27}{30} \div 8\frac{1}{3}$

8)  $1\frac{7}{9} \div 1\frac{2}{3}$

9)  $4\frac{2}{3} \div 3\frac{1}{2}$

10)  $2\frac{3}{11} \div 3\frac{9}{22}$

b) Find the following:

1)  $\frac{4}{5}$  divided by  $\frac{2}{5}$

2)  $\frac{2}{5}$  divided by  $\frac{4}{5}$

3)  $\frac{9}{4}$  into  $\frac{12}{25}$

4) Divide  $1\frac{2}{3}$  by  $\frac{2}{3}$

5)  $\frac{4}{9}$  into  $1\frac{4}{5}$

6) Divide  $\frac{1}{4}$  by  $\frac{2}{5}$

**FRACTION REVIEW:**

Decide which operation (+, -, x, ÷) by the wording in the question. Then find the answer.

1) What is  $\frac{3}{5}$  of 40?

2) How much is  $4\frac{2}{3}$  from  $6\frac{1}{5}$ ?

3) How much is  $\frac{1}{2}$  from  $\frac{2}{3}$ ?

4) How much is  $\frac{3}{7}$  of 21?

5)  $\frac{4}{5}$  and  $\frac{2}{3}$  equals....?

6)  $\frac{2}{3}$  divided by 14 is what number?

7) What is  $\frac{9}{10}$  of 300?

8) What is  $\frac{2}{5}$  into 12?

9)  $\frac{3}{4}$  from  $\frac{8}{9}$  equals....?

10) What is  $1\frac{2}{3}$  by  $3\frac{3}{8}$ ?

11) How much is  $\frac{5}{7}$  of  $\frac{21}{50}$ ?

12) Find the total of  $\frac{2}{3}$  and  $\frac{1}{6}$  and  $\frac{4}{9}$ ?

13)  $\frac{3}{4}$  of  $\frac{2}{3}$  equals....?

14) What is  $\frac{2}{5}$  greater than  $4\frac{3}{5}$ ?

**ANSWER KEY – FRACTION REVIEW****EXERCISE 1: Introduction to Fractions (Page 3)**

a) 1)  $\frac{4}{6}$  2)  $\frac{9}{12}$  3)  $\frac{25}{40}$  4)  $\frac{2}{32}$  5)  $\frac{6}{45}$  6)  $\frac{21}{27}$  7)  $\frac{70}{100}$  8)  $\frac{33}{44}$

b) 1)  $\frac{2}{4}$  2)  $\frac{8}{9}$  3)  $\frac{3}{5}$  4)  $\frac{5}{7}$  5)  $\frac{4}{6}$  6)  $\frac{45}{50}$

c) 1)  $\frac{3}{4}$  2)  $\frac{2}{3}$  3)  $\frac{3}{4}$  4)  $\frac{3}{4}$  5)  $\frac{4}{5}$  6)  $\frac{2}{3}$  7)  $\frac{1}{2}$  8)  $\frac{2}{3}$  9)  $\frac{2}{3}$  10)  $\frac{3}{5}$

**EXERCISE 2: Types of Fractions (Page 6)**

a) 1) C 2) M 3) I 4) W 5) W 6) M 7) W 8) I 9) C 10) W

b) 1)  $1\frac{2}{5}$  2)  $1\frac{7}{11}$  3)  $1\frac{9}{61}$  4)  $2\frac{2}{5}$  5)  $1\frac{1}{99}$  6)  $12\frac{1}{2}$

c) 1)  $\frac{11}{5}$  2)  $\frac{51}{8}$  3)  $\frac{26}{3}$  4)  $\frac{56}{5}$  5)  $\frac{49}{5}$  6)  $\frac{19}{4}$

d) 1)  $\frac{7}{10}$  2) 8 3)  $2\frac{2}{3}$  4)  $5\frac{1}{2}$  5)  $\frac{5}{3}$  or  $1\frac{2}{3}$  6)  $\frac{15}{2}$  or  $7\frac{1}{2}$

**EXERCISE 3: Comparing Fractions (Page 9)**

1)  $\frac{7}{13}$  2)  $1\frac{9}{10}$  3)  $\frac{9}{10}$  4)  $\frac{5}{12}$  5)  $\frac{5}{8}$  6)  $\frac{7}{12}$  7)  $\frac{11}{15}$  8)  $\frac{4}{9}$  9)  $\frac{1}{4}$

**EXERCISE 4: Adding Fractions (Page 12)**

a) 1)  $\frac{3}{5}$  2)  $\frac{7}{5}$  or  $1\frac{2}{5}$  3)  $\frac{2}{3}$  4)  $\frac{9}{4}$  or  $2\frac{1}{4}$

b) 1)  $\frac{7}{9}$  2)  $\frac{7}{8}$  3)  $\frac{9}{16}$  4)  $\frac{14}{15}$

c) 1)  $7\frac{3}{4}$  2)  $12\frac{5}{6}$  3)  $13\frac{3}{10}$  4)  $9\frac{1}{4}$  5)  $11\frac{1}{6}$  6)  $15\frac{1}{12}$  7)  $8\frac{7}{15}$  8)  $16\frac{7}{24}$

d) 1)  $1\frac{2}{21}$  2)  $1\frac{5}{24}$  3)  $\frac{7}{10}$  4)  $2\frac{3}{14}$  5)  $\frac{19}{24}$  6)  $\frac{8}{21}$

**EXERCISE 5: Subtracting Fractions (Page 14)**

- a) 1)  $\frac{5}{8}$     2)  $\frac{23}{30}$     3)  $\frac{11}{24}$     4)  $\frac{1}{9}$     5)  $3\frac{1}{2}$     6)  $3\frac{1}{4}$     7)  $\frac{1}{8}$     8)  $\frac{1}{4}$
- b) 1)  $3\frac{2}{3}$     2)  $7\frac{1}{2}$     3)  $\frac{6}{7}$     4)  $2\frac{5}{6}$     5)  $10\frac{11}{12}$     6)  $4\frac{19}{24}$
- c) 1)  $\frac{7}{16}$     2)  $\frac{5}{21}$     3)  $\frac{1}{3}$     4)  $\frac{5}{24}$

**EXERCISE 6: Multiplying Fractions (Page 16)**

- a) 1)  $\frac{1}{2}$     2)  $\frac{2}{3}$     3) 1    4)  $\frac{2}{7}$     5)  $\frac{6}{25}$     6)  $\frac{1}{4}$     7) 6000
- 8) 10    9)  $\frac{130}{21}$  or  $6\frac{4}{21}$     10)  $1\frac{1}{2}$     11) 3    12)  $\frac{29}{2}$  or  $14\frac{1}{2}$
- b) 1) 30    2) 82    3)  $\frac{3}{2}$  or  $1\frac{1}{2}$     4)  $\frac{12}{7}$  or  $1\frac{5}{7}$     5)  $\frac{31}{63}$

**EXERCISE 7: Dividing Fractions (Page 18)**

- a) 1) 1    2)  $\frac{5}{28}$     3) 9    4)  $\frac{1}{24}$     5)  $\frac{8}{9}$     6)  $4\frac{1}{2}$     7)  $\frac{27}{250}$
- 8)  $1\frac{1}{15}$     9)  $1\frac{1}{3}$     10)  $\frac{2}{3}$
- b) 1) 2    2)  $\frac{1}{2}$     3)  $\frac{16}{75}$     4)  $2\frac{1}{2}$     5)  $4\frac{1}{20}$     6)  $\frac{5}{8}$

**Fraction Review (Page 19)**

- a) 1) 24    2)  $1\frac{8}{15}$     3)  $\frac{1}{6}$     4) 9    5)  $1\frac{7}{15}$     6)  $\frac{1}{21}$     7) 270
- 8) 30    9)  $\frac{5}{36}$     10)  $5\frac{5}{8}$     11)  $\frac{3}{10}$     12)  $1\frac{5}{18}$     13)  $\frac{1}{2}$     14) 5

# DECIMALS

**This material is intended as a review of skills you once learned and wish to study before your assessment.**

You should be familiar with all of the topics included here. Read the explanations carefully, and do as many of each type of question in each exercise until you have mastered the material.

Check your answers in the answer key at the end.

Do **not** use a calculator, unless you are instructed to.

*ANSWER KEY AT THE END*

## DECIMAL REVIEW

### A. INTRODUCTION TO THE DECIMAL SYSTEM

The Decimal System is another way of expressing a part of a whole number. A decimal is simply a fraction with a denominator of 10, 100, 1 000 or 10 000 etc. The number of decimal places refers to how many zeros will be in the denominator. Note that the number 5.62 is read as five point six two.

The <u>first</u> decimal place refers to <u>tenths</u>	$2.3 = 2\frac{3}{10}$
The <u>second</u> decimal place refers to <u>hundredths</u>	$2.31 = 2\frac{31}{100}$
The <u>third</u> decimal place refers to <u>thousandths</u>	$2.319 = 2\frac{319}{1000}$

Similarly, six decimal places would be a fraction with a denominator of 1 000 000 (millionths). The most common usage of decimals is in our monetary system where 100 cents (2 decimal places) make up one dollar. For example, \$2.41 is really two dollars and forty-one hundredths ( $\frac{41}{100}$ ) of a dollar.

#### Examples:

1. **Change 2.30 to a fraction**  
 Notice that 2.30 is the same as 2.3  
 In fact,  $2.30 = 2.300 = 2.3000$  etc.

$2.30 = 2\frac{30}{100} = 2\frac{3}{10}$
2. **Change 0.791 to a fraction**  
 Notice that  $0.791 = .791$   
 The zero in front of the decimal place is not needed.

$0.791 = \frac{791}{1000}$
3. **Change .003 to a fraction**  
 Notice that the zeros in this example are important.

$.003 = \frac{3}{1000}$
4. **Simplify 0.0024000**  
 Notice that zero at the end or zero as a whole number (to the left of the decimal) is not needed.

$0.0024000 = .0024$

**B. OPERATIONS WITH DECIMALS****1. Addition:****Add \$2.50 and \$1.35**

Place numbers in columns, so that the decimal places are in line. Place the decimal point in the same line for the answer.

$$\begin{array}{r} \$2.50 \\ + \$1.35 \\ \hline \$3.85 \end{array}$$

Now, add as if adding whole numbers.

**Add 1.3928 and 12.43 and .412**

Add zeros to fill in columns. This will not change the value of the number (see examples on previous page.)

$$\begin{array}{r} 1.3928 \\ + 12.4300 \\ + 0.4120 \\ \hline 14.2348 \end{array}$$

**2. Subtraction:****Subtract \$1.30 from \$5.45**

Set up columns as in addition, and subtract as if subtracting whole numbers.

$$\begin{array}{r} \$5.45 \\ - \$1.30 \\ \hline \$4.15 \end{array}$$

**Calculate 5 minus .2982**

Fill in columns with zeros, as in addition.

*Note the difference in wording in these two examples.*

$$\begin{array}{r} 5.0000 \\ - 0.2982 \\ \hline 4.7018 \end{array}$$

**3. Multiplication:****Multiply 2.12 by 4.2**

At first, ignore the decimals, and multiply as if calculating  $212 \times 42$ .

Now, add up the decimal places in both numbers and your answer will have that total number of decimal places.

$$\begin{array}{r} 2.12 \text{ (2 places)} \\ \times 4.2 \text{ (1 place)} \\ \hline 424 \\ 8480 \\ \hline 8.904 \text{ (3 places)} \end{array}$$

**Product of 0.0941 and .02**

Multiply. Add enough zeros to show the correct number of decimal places

$$\begin{array}{r} .0941 \text{ (4 places)} \\ \times .02 \text{ (2 places)} \\ \hline .001882 \text{ (total = 6)} \end{array}$$

**Multiply** .5624 by 1000

Notice  $.5624 \times 1000 = 562.4$ , so multiplying  $\times 1000$  (0 decimal places) is the same as moving the decimal place 3 places (since 1000 has 3 zeros) to the right.

$$\begin{array}{r} .5624 \quad (4 \text{ places}) \\ \times 1000 \quad (0 \text{ places}) \\ \hline 562.4000 \quad (\text{total} = 4) \end{array}$$

Similarly,

$$\begin{array}{l} .58 \times 10 = 5.8 \quad 5.636 \times 10\,000 = 56\,360 \\ .58 \times .1 = .058 \quad .58 \times .001 = .00058 \end{array}$$

or 562.4

**Division:** Divide 2.322 by .12

The divisor (.12) must be changed to a whole number (12) by moving the decimal point 2 places to the right, in both numbers. In the answer, place the decimal point directly above the numbers. Note that zeros must be added to complete the division.

$$\begin{array}{r} .12 \overline{)2.322} \\ \underline{12} \phantom{0} \\ 12 \phantom{0} \\ \underline{108} \\ 42 \\ \underline{36} \\ 60 \\ \underline{60} \\ 0 \end{array}$$

Divide .003 into 51

Firstly, notice the difference in wording in these two division questions. Zero must be added to the number 51 in order to move the decimal 3 places to the right.

$$\begin{array}{r} .003 \overline{)51} \\ \underline{3} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

Divide 254.25 by 1000

Notice  $254.25 \div 1000 = .25425$ . So, dividing by 1000 is the same as moving the decimal 3 places to the left (since 1000 has 3 zeros).

$$\begin{array}{r} 1000 \overline{)254.250} \\ \underline{2000} \\ 5425 \\ \underline{5000} \\ 4250 \\ \underline{4000} \\ 2500 \\ \underline{2000} \\ 5000 \\ \underline{5000} \\ 0 \end{array}$$

Similarly,  $32.52 \div 10 = 3.252$   
 $2.6 \div 10\,000 = .00026$

**EXERCISE 1: Decimal Operations** Do not use a calculator.**a) Change to fractions: (remember to reduce)**

- 1) 5.8      2) 27.3400      3) 30.02      4) 0.590      5) 3.075

**b) Addition:**

- 1)  $2.49 + .32$       2) 0.042 plus .00982      3) 7.342 and 2 and 7.65  
4)  $1743.2 + 2.984 + 12.35$       5) 2.76 more than 8.4590

**c) Subtraction:**

- 1) 2.036 from 4.478      2) 12.258 from 13      3) 670.1 minus 589.213  
4) 0.1002 minus 0.05      5) 19.6 decreased by 5.349

**d) Multiplication:**

- 1) .21 by .04      2) .42 x .218      3) .75 times 132.786  
4) Product of .009 and 2.003      5) .25 of 288  
6) 9.4325 by 1000      7) 9.4325 by .001

**e) Division:**

- 1) 248 divided by 0.8      2) 15.47 divided by .091  
3) 40.4 into 828.2      4) 0.0338 divided by 1.30  
5) .0025 into 1.875      6) 923.56 divided by 1000  
7) 923.56 divided by .01

### C. CONVERSION AND ROUNDING OFF

#### 1. Converting Decimals to Fractions:

When changing decimals to fractions, simply create a fraction with 10, 100, 1000 etc. in the denominator. The number of zeros in the denominator is the same as the number of decimal places.

$$0.913 = \frac{913}{1000}$$

$$5.25 = 5\frac{25}{100} = 5\frac{1}{4}$$

#### 2. Converting Fractions to Decimals:

When changing a fraction to a decimal, simply divide the denominator into the numerator. Every time a fraction is changed to a decimal, the division will either stop (as in  $\frac{4}{5} = 0.8$ ) or the division will go on forever

$$\frac{4}{5} = 4 \div 5 = 0.8$$

by repeating (as in  $\frac{2}{3} = \dot{.}6$ ). Notice that a repeating decimal is shown by a dot over the number (if only one number repeats) or as a bar (if more than one number repeats, as in  $\frac{2}{11} = \overline{.18}$ )

$$\frac{2}{3} = 2 \div 3 = 0.666\dots = \dot{.}6$$

$$\frac{2}{11} = 2 \div 11 = .1818\dots = \overline{.18}$$

#### 3. Repeating Decimals

Following is a list of some repeating decimals:

$$\frac{1}{3} = \dot{.}3$$

$$\frac{1}{6} = .1\dot{6}$$

$$\frac{1}{7} = \overline{.142857}$$

$$\frac{2}{3} = \dot{.}6$$

$$\frac{5}{6} = \dot{.}8\dot{3}$$

$$\frac{1}{9} = \overline{.11} \text{ or } \dot{.}1$$

$$\frac{2}{9} = \overline{.22} \text{ or } \dot{.}2$$

$$\frac{4}{9} = \overline{.44} \text{ or } \dot{.}4$$

$$\frac{5}{9} = \overline{.55} \text{ or } \dot{.}5$$

$$\frac{7}{9} = \overline{.77} \text{ or } \dot{.}7$$

$$\frac{8}{9} = \overline{.88} \text{ or } \dot{.}8$$

#### 4. Rounding off:

If we want to divide \$2.00 into 3 equal parts, we would want our answer to be to the nearest cent (or nearest hundredth). Since our answer is closer to 67 cents than 66 cents, we would round off our answer to \$0.67.

$$\begin{array}{r} 0.666\dots \\ 3 \overline{)2.000} \end{array}$$

When rounding to the nearest thousandth, we want our answer to have 3 decimal places. If the fourth decimal place has a 5 or greater, round up. If less than a 5, do not round up.

$$2.6549 \text{ to nearest thousandth} = 2.655$$

$$2.6549 \text{ to nearest hundredth} = 2.65$$

$$3.95 \text{ to nearest tenth} = 4.0$$

**EXERCISE 2: Conversion and Rounding Off****a) Change to Fractions: (Remember to reduce whenever possible)**

1. 2.591      2. 25.030      3. 50.0250      4. 0.8

**b) Change to Decimals: (Do not round off)**

1.  $\frac{7}{8}$       2.  $\frac{3}{11}$       3.  $\frac{4}{9}$       4.  $\frac{3}{5}$       5.  $\frac{4}{7}$   
6.  $\frac{5}{12}$       7.  $\frac{5}{6}$       8.  $\frac{2}{3}$       9.  $\frac{7}{2}$       10.  $\frac{2}{9}$

**c) Round Off:**

1. 2.864 to nearest hundredth      2. 35.9649 to nearest thousandth  
3. 931.85 to nearest tenth      4. 2.091 to nearest tenth  
5. 11.898 to nearest hundredth      6. 12.92 to nearest whole number  
7. 11.74235 to nearest thousandth      8.  $\frac{5}{9}$  to nearest thousandth  
9.  $\frac{7}{12}$  to nearest hundredth

<b>ANSWERS</b>
----------------

**EXERCISE 1: Decimal Operations**

- |                      |                      |                     |                     |                    |
|----------------------|----------------------|---------------------|---------------------|--------------------|
| a) 1) $5\frac{4}{5}$ | 2) $27\frac{17}{50}$ | 3) $30\frac{1}{50}$ | 4) $\frac{59}{100}$ | 5) $3\frac{3}{40}$ |
| b) 1) 2.81           | 2) .05182            | 3) 16.992           | 4) 1758.534         | 5) 11.219          |
| c) 1) 2.442          | 2) .742              | 3) 80.887           | 4) .0502            | 5) 14.251          |
| d) 1) .0084          | 2) .09156            | 3) 99.5895          | 4) .018027          | 5) 72              |
| 6) 9432.5            | 7) .0094325          |                     |                     |                    |
| e) 1) 310            | 2) 170               | 3) 20.5             | 4) .026             | 5) 750             |
| 6) .92356            | 7) 92356             |                     |                     |                    |

**EXERCISE 2: Conversion and Rounding Off**

- |                           |                      |                     |                  |                         |
|---------------------------|----------------------|---------------------|------------------|-------------------------|
| a) 1) $2\frac{591}{1000}$ | 2) $25\frac{3}{100}$ | 3) $50\frac{1}{40}$ | 4) $\frac{4}{5}$ |                         |
| b) 1) .875                | 2) $\overline{.27}$  | 3) $\dot{.4}$       | 4) .6            | 5) $\overline{.571428}$ |
| 6) $\dot{.416}$           | 7) $\dot{.83}$       | 8) $\dot{.6}$       | 9) 3.5           | 10) $\dot{.2}$          |
| c) 1) 2.86                | 2) 35.965            | 3) 931.9            | 4) 2.1           | 5) 11.90                |
| 6) 13                     | 7) 11.742            | 8) .556             | 9) .58           |                         |

# PERCENTS

**This material is intended as a review of skills you once learned and wish to study before your assessment.**

You should be familiar with all of the topics included here. Read the explanations carefully, and do as many of each type of question in each exercise until you have mastered the material.

Check your answers in the answer key at the end.

Do **not** use a calculator, unless you are instructed to.

*ANSWER KEY AT THE END*

**PERCENT REVIEW**

The use of percentage is another way of expressing numbers (usually fractions) in such a way as to make comparisons between them more obvious. For instance, if you get 28 out of 40 in test A and 37 out of 50 in Test B, it may not be clear whether you have improved or not. The use of percentage will allow this comparison, because a percent is part of 100. (i.e. a percent is a fraction with a denominator of 100).

**A. CHANGING % TO FRACTIONS / DECIMALS**

A percent means a part of 100. For example, if you get 95% on a test, your mark was 95 out of 100. A percent can be changed to a fraction or decimal by simply dividing the percentage number by 100.

**1. Changing % to Fractions**

Divide by 100.

(i.e. put the % number over 100 and reduce if necessary)

$$93\% = \frac{93}{100}$$

$$50\% = \frac{50}{100} = \frac{1}{2}$$

$$24\% = \frac{24}{100} = \frac{6}{25}$$

If a decimal appears in the fraction, multiply the fraction by 10, 100, 1000 etc. to produce an equivalent fraction without decimals.

$$26.3\% = \frac{26.3}{100} \times \frac{10}{10} = \frac{263}{1000}$$

$$5.55\% = \frac{5.55}{100} \times \frac{100}{100} = \frac{555}{10000} = \frac{111}{2000}$$

**2. Changing % to Decimals**

Simply divide by 100. (i.e. move the decimal point 2 places to the left)

$$50\% = .50 \quad \text{or} \quad .5$$

$$9.23\% = .0923$$

$$4\% = .04$$

$$148\% = 1.48$$

**B. CHANGING FRACTIONS / DECIMALS TO PERCENTS**

**1. Changing Fractions to Percent**

If you get 17 out of 20 on a test, it is convenient to change this mark to a percentage.

This means changing  $\frac{17}{20}$  to an equivalent fraction with 100 as denominator (i.e.  $\frac{17}{20} = \frac{?}{100}$ ).

To change fractions to %, simply multiply the fraction by 100%.

$$\frac{17}{20} = \frac{17}{20} \times 100\% = \frac{1700}{20} = 85\%$$

$$\frac{1}{2} = \frac{1}{2} \times 100\% = 50\%$$

$$\frac{2}{3} = \frac{2}{3} \times 100\% = 66.\dot{6}\%$$

$$\frac{19}{40} = \frac{19}{40} \times 100\% = 47\frac{1}{2}\% \text{ or } 47.5\%$$

Note: The mathematical wording for changing a fraction ( $\frac{17}{20}$ ) to a percent would normally be:

**17 is what % (out) of 20?  
or  
What % is 17 (out) of 20?**

The word "out" is usually not included.

e.g. 19 is what % of 75?

$$\frac{19}{75} = \frac{19}{75} \times 100\% = 25\frac{1}{3}\%$$

What % is 7 of 5?

$$\frac{7}{5} = \frac{7}{5} \times 100\% = 140\%$$

**2. Changing Decimals to Percents**

To change decimals to percents, simply multiply by 100% (i.e. move the decimal point 2 places to the right.)

$$\begin{aligned} .29 &= .29 \times 100\% = 29\% \\ .156 &= .156 \times 100\% = 15.6\% \\ 1.3 &= 1.3 \times 100\% = 130\% \end{aligned}$$

C. USING PERCENTS

When percents are used in calculations, they are first converted to either fractions or decimals. Usually it is more convenient to change % to decimals.

1. Multiplying With Percents

If a test mark was 50% and it was out of 40 total marks, what was the test score?

$$50\% = \frac{\text{test score?}}{40 \text{ (total marks)}}$$

$$50\% \times 40 = .5 \times 40 = 20$$

$$\text{So } 50\% = \frac{20 \text{ marks}}{40}$$

**50% (out) of 40 is what number?**  
**What number is 50% (out) of 40?**

To find the test score, or the part, we multiply the % by the total.

e.g. 85% of 25 is what number?  $85\% \times 25 = .85 \times 25 = 21.25$

What number is 30% of 45.37?  $30\% \times 45.37 = .3 \times 45.37 = 13.611$

2. Dividing with Percents

If a test mark was 50% and you received a score of 20 marks, what was the test out of?

$$50\% = \frac{20 \text{ marks}}{\text{total?}}$$

$$20 \div 50\% = 20 \div .5 = 40$$

$$\text{or } 20 \div \frac{1}{2} = 20 \times \frac{2}{1} = 40$$

$$\text{So, } 50\% = \frac{20}{40} \text{ total marks}$$

**50% (out) of what number is 20?**  
**20 is 50% (out) of what number?**

To find the total marks, we divide by the %.

e.g. 40% of what number is 25?  $25 \div 40\% = 25 \div .40 = 62.5$

18 is 75% of what number?  $18 \div 75\% = 18 \div .75 = 24$

**D. SUMMARY AND EXERCISE**

**1. Three types of Percent Problems**

In summary, there are three things that we can do with percent. We will use the example on the right side of the page to summarize.

$$50\% = \frac{20 \text{ (part)}}{40 \text{ total}}$$

1. Finding % or what % of 40 is 20?

$$\frac{20}{40} = \frac{20}{40} \times 100\% = 50\%$$

2. Finding the Part or 50% of 40 is what number?

$$50\% \times 40 = .5 \times 40 = 20$$

3. Finding the total or 50% of what number is 20?

$$20 \div 50\% = 20 \div .5 = 40$$

**2. EXERCISE: PERCENT PROBLEMS**

**1. Change to Fractions**

- |          |        |                     |          |
|----------|--------|---------------------|----------|
| a) 97%   | b) 82% | c) 150%             | d) 45.3% |
| e) 9.25% | f) 40% | g) $5\frac{1}{2}\%$ |          |

**2. Change to Decimals**

- |          |          |       |           |
|----------|----------|-------|-----------|
| a) 42%   | b) 9.37% | c) 2% | d) 243.9% |
| e) 0.95% |          |       |           |

**3. Change to %**

- |                    |                    |                    |                   |
|--------------------|--------------------|--------------------|-------------------|
| a) $\frac{19}{20}$ | b) $\frac{2}{3}$   | c) $\frac{18}{75}$ | d) $\frac{1}{12}$ |
| e) $\frac{5}{9}$   | f) $\frac{38}{40}$ | g) 0.865           | h) 2.37           |
| i) .0092           | j) $\frac{7}{4}$   |                    |                   |

**4. Finding %**

- a) What % of 72 is 18?
- b) 16 is what % of 80?
- c) What % of 30 is 18.5?

**5. Finding the Part**

- a) 40% of 18 is what number?
- b) What number is 16.5% of 30.2?
- c) 65% of 15 is what?

**6. Finding the Total**

- a) 40% of what number is 12?
- b) 18 is 55% of what number?
- c) 120 is 150% of what number?

**7. Percent Problems Combined**

- a) What % of 25 is 5?
- b) 70% of 15 is what number?
- c) 85 is 20% of what number?
- d) 90 is what % of 55?
- e) 30% of what number is 80?
- f) What number is 42% of 50?

**ANSWERS**

1. a)  $\frac{97}{100}$     b)  $\frac{41}{50}$     c)  $1\frac{1}{2}$     d)  $\frac{453}{1000}$     e)  $\frac{37}{400}$     f)  $\frac{2}{5}$     g)  $\frac{11}{200}$
2. a) .42    b) .0937    c) .02    d) 2.439    e) .0095
3. a) 95%    b)  $66.\dot{6}\%$  or  $66\frac{2}{3}\%$     c) 24%    d)  $8.\dot{3}\%$     e)  $55.\dot{5}\%$     f) 95%
- g) 86.5%    h) 237%    i) .92%    j) 175%
4. a) 25%    b) 20%    c)  $61.\dot{6}\%$
5. a) 7.2    b) 4.983    c) 9.75
6. a) 30    b)  $32.\overline{72}$     c) 80
7. a) 20%    b) 10.5    c) 425    d)  $163.\overline{63}\%$     e)  $266.\dot{6}$     f) 21



## EXPONENTS AND ROOTS

### A. EXPONENTS

In math, many symbols have been developed to simplify certain types of number expressions. One of these symbols is the “exponent”.

**Rule: An exponent indicates how many times a base number is used as a factor.**

Example:  $10^4 = 10 \times 10 \times 10 \times 10$  or  $(10)(10)(10)(10)$   
 $3^5 = 3 \times 3 \times 3 \times 3 \times 3$   
 $x^2 = (x)(x)$

The exponent is written smaller and is placed above the base number (the number to be multiplied). The first example can be read “ten exponent four” or “ten to the power of four”.

Second and third power have special names: second power is usually called “squared”, and third power is usually called “cubed”.

Example:  $5^2$  is “five squared”  
 $5^3$  is “five cubed”

A simple way to work out exponents is to write the base digit the same number of times as the value of the exponent, and put a multiplication sign between each digit.

Example:  $4^5 = 4 \times 4 \times 4 \times 4 \times 4$  (the exponent is 5, so write 4 five times)  
 $= 1024$

### PRACTICE A:

Write as an exponent or power (also called “*exponential notation*”):

1.)  $5 \cdot 5 \cdot 5 \cdot 5 =$

2.)  $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 =$

3.)  $10 \cdot 10 =$

4.)  $6 \cdot 6 \cdot 6 =$

Evaluate the following:

5.)  $2^4 =$

6.)  $5^3 =$

7.)  $3^4 =$

8.)  $2^5 =$

9.)  $10^2 =$

10.)  $1^7 =$

11.)  $\left(\frac{1}{4}\right)^2 =$

12.)  $\frac{4^2}{5} =$

13.)  $\left(\frac{4}{5}\right)^2 =$

14.)  $10^3 + 4^2 =$

**B. SQUARE ROOTS**

**Rule: Taking the square root of a number is the reverse of squaring the number.**

The symbol for square root is  $\sqrt{\quad}$

Example:  $\sqrt{9}$  (the square root of 9) = 3 because  $(3)^2 = 9$   
 $\sqrt{144}$  (the square root of 144) = 12 because  $(12)^2 = 144$

The square of a number can be positive (+) or negative (-). This is because two positive numbers multiplied together or two negative numbers multiplied together make a positive number.

However, the  $\sqrt{\quad}$  symbol tells us we should write only the positive root.

Example:  $\sqrt{4} = 2$ , but remember,  $(+2)^2 = 4$  or  $(-2)^2 = 4$   
 $\sqrt{25} = 5$ , but either  $(+5)^2 = 25$  or  $(-5)^2 = 25$

**PRACTICE B:**

Evaluate the following:

- |                               |                                |
|-------------------------------|--------------------------------|
| 1.) $\sqrt{4} =$              | 2.) $\sqrt{1} =$               |
| 3.) $\sqrt{121} =$            | 4.) $\sqrt{81} =$              |
| 5.) $\sqrt{100} =$            | 6.) $\sqrt{10000} =$           |
| 7.) $\frac{\sqrt{36}}{49} =$  | 8.) $\sqrt{\frac{36}{49}} =$   |
| 9.) $\frac{\sqrt{81}}{100} =$ | 10.) $\sqrt{\frac{81}{100}} =$ |

**ANSWERS****Practice A (exponents):**

- |   |   |   |
|---|---|---|
| 1.) $5^4$   | 2.) $2^5$   | 3.) $10^2$  |
| 4.) $6^3$   | 5.) $2 \times 2 \times 2 \times 2 = 16$               | 6.) $5 \times 5 \times 5 = 125$                           |
| 7.) $3 \times 3 \times 3 \times 3 = 81$   | 8.) $2 \times 2 \times 2 \times 2 \times 2 = 32$      | 9.) $10 \times 10 = 100$                                  |
| 10.) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$              | 11.) $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$  | 12.) $\frac{4 \times 4}{5} = \frac{16}{5} = 3\frac{1}{5}$ |
| 13.) $\left(\frac{4}{5}\right) \times \left(\frac{4}{5}\right) = \frac{16}{25}$ | 14.) $(10 \times 10 \times 10) + (4 \times 4) = 1016$ |   |

**Practice B (square roots)**

- |         |                    |                   |                     |                     |
|---------|--------------------|-------------------|---------------------|---------------------|
| 1.) 2   | 2.) 1              | 3.) 11            | 4.) 9               | 5.) 10              |
| 6.) 100 | 7.) $\frac{6}{49}$ | 8.) $\frac{6}{7}$ | 9.) $\frac{9}{100}$ | 10.) $\frac{9}{10}$ |